

11/28/16

"Remember no one can make you feel inferior without your consent." -Eleanor Roosevelt

HW: "Implicit Differentiation" w/s #1-6

Test 1 on Thursday 12/1

AIM: How do we do Implicit Differentiation?

Warm-up:

Find  $\frac{dy}{dx}$  for each of the following:

1)  $y = 3x + 5$

$$\frac{dy}{dx} = 3$$

2)  $xy = 5$

$$y = \frac{5}{x} = 5x^{-1}$$

$$\frac{dy}{dx} = -5x^{-2}$$

$$\frac{dy}{dx} = -\frac{5}{x^2}$$

For these,  $y$  is a function explicitly defined in terms of  $x$ .

$$y = 3x^2 + 2x$$

$$y = x + 1$$

$$y = \sin x$$

$$y = \sqrt{x^2 + 2}$$

For these,  $y$  is a function implicitly defined in terms of  $x$ .

$$xy = 12$$

$$3x^2y + 2y^2 - 5 = 0$$

$$x + xy + y = 5$$

$$y = \cos(xy)$$

Be able to explain what makes a function implicitly defined vs. explicitly defined.

$y$  is mixed  
in with " $x$ "s

$y$  is by itself

$$\frac{dy}{dx} = \text{"Derivative of } y \text{ with respect to } x\text{"}$$

## Implicit Differentiation

$$\text{ex. } \frac{d}{dx}[y] = \boxed{\frac{dy}{dx}}$$

$$\text{ex. } \frac{d}{dx}[2y] = 2 \cdot \frac{dy}{dx}$$

$$\text{ex. } \frac{d}{dx}[y^2] = 2y \cdot \frac{dy}{dx}$$

$$\frac{d}{dx} [x^3 + 2x]$$

Think:

$$f(x) = 2x$$

$$f'(x) = 2$$

Implicit Differentiation

Take the derivative of each term with respect to  $x$  and solve for  $dy/dx$ .

ex. Find  $dy/dx$  for  $(y)^2 + 3x = x^2$

get  $\frac{dy}{dx}$  by itself

$$2(y) \cdot \frac{dy}{dx} + 3 = 2x$$

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$$\frac{2y \cdot \frac{dy}{dx}}{2y} = \frac{2x - 3}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{2x - 3}{2y}}$$

ex. Find dy/dx for  $\sin x + \cos y = 2y$

$$\begin{aligned} \cos(x) - \sin(y) \cdot \frac{dy}{dx} &= 2 \frac{dy}{dx} \\ &+ \sin(y) \frac{dy}{dx} \quad + \sin(y) \frac{dy}{dx} \\ \hline \cos(x) &= 2 \cdot \frac{dy}{dx} + \sin(y) \cdot \frac{dy}{dx} \\ \frac{\cos(x)}{2 + \sin(y)} &= \frac{\frac{dy}{dx} (2 + \sin(y))}{2 + \sin(y)} \\ \boxed{\frac{dy}{dx} = \frac{\cos(x)}{2 + \sin(y)}} \end{aligned}$$

ex. Find dy/dx for  $y^2 + \boxed{xy} + x^2 = 5$  product rule

$$\begin{aligned} 2y \cdot \frac{dy}{dx} + \boxed{x \cdot \frac{dy}{dx} + 1y} + 2x &= 0 \\ &\quad -1y \quad -2x \quad -1y - 2x \\ \hline 2y \cdot \frac{dy}{dx} + x \frac{dy}{dx} &= -1y - 2x \\ \frac{dy}{dx} (2y + x) &= \frac{-1y - 2x}{2y + x} \\ \boxed{\frac{dy}{dx} = \frac{-1y - 2x}{2y + x}} \end{aligned}$$

ex.  $x^2 - xy + y^2 = 3$

a. Show that  $\frac{dy}{dx} = \frac{y - 2x}{2y - x}$

b. Find any point(s) where the tangent line is horizontal.